Progress towards a 4D Structured Multiblock Grid Generator

Ray Hixon

Mechanical, Industrial, and Manufacturing Engineering (MIME) Department
University of Toledo

Duane.Hixon@utoledo.edu

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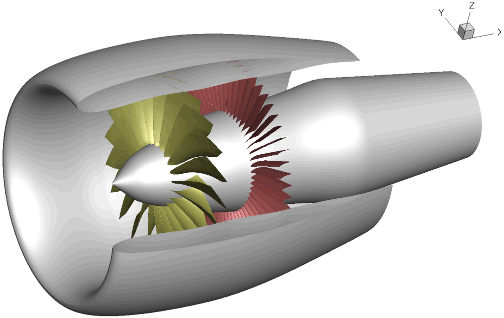
Background

- The goal: High-accuracy predictions of unsteady flow and noise in complex jet engine geometries...
- The idea: Solving the problem in 4D space-time provides new efficiencies for unsteady flow prediction.



A revolutionary CAA prediction tool is being developed at NASA

The Advantages of Structured Multiblock Methods in Space-Time

- Some of the practical advantages of the space-time approach are:
  - Can use a *lot* of existing structured grid technology:
    - Robust and efficient CAA differencing schemes.
    - Fast CFD steady-flow convergence techniques.
    - Smooth elliptic grid generation methods.
  - This also sidesteps a *lot* of existing problems with time marching:
    - How to resolve a range of time scales?
    - How to transfer data efficiently between moving and stationary grids?
    - How to achieve large-scale parallel efficiency?
  - Gives a pathway to quantum computing...

The space-time method could solve a lot of problems!

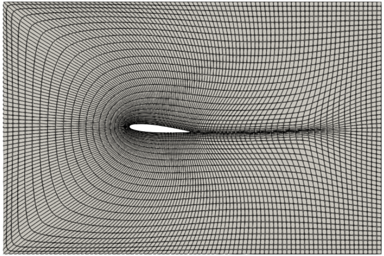
The Challenges of Structured Multiblock Methods in Space-Time

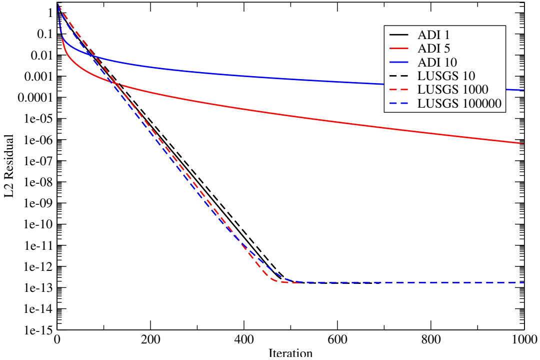
- There are a *lot* of questions that must be answered! For example:
  - Structured grid generation:
    - Do the existing 3D CFD convergence methods work for 4D elliptic equations?
    - Need **automatic structured** grid generation for arbitrary configurations **in 4D!**
    - Need **automatic grid adaption** to the flow solution **in 4D!**
  - Flow solver:
    - Do the existing implicit 3D CFD convergence methods work for high-accuracy differencing methods?
    - How to maintain accuracy on complex grids with grid singularities?
    - What are the 4D flow boundary conditions?

There are a lot of obstacles that must be overcome on the way!

Grid Generation in 4D (January 2023)

- A 4D single-block C-grid was generated about a pitching NACA 0012 infinite wing.
- Both the ADI and LU-SGS methods successfully converged the grid.

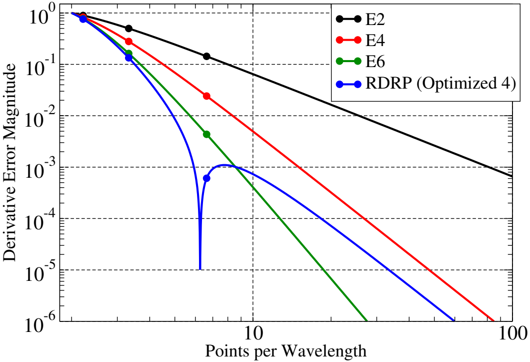




The grid generation equations were converged to machine precision!

Reducing Computational Effort...

- High-accuracy numerical differencing schemes are necessary for computational efficiency.
- All of these schemes are implemented in the Space-Time Mapping Analysis (STMA) code.



A range of numerical schemes are being tested!

Generalized Four Dimensional STMA Flow Solver (March 2025)

- Inviscid Single Block Solver:
  - Method of Manufactured Solution (MMS) Verification
  - Test Case: Pitching NACA Airfoil at $M = 0.1$
  - 4D 'scaled' LU-SGS method: high order RHS with 'scaled' low-order LHS
  - Shock-capturing artificial dissipation (extension of JST (1981) approach)
- Accomplishments:
  - MMS used to Verify RHS implementation and order of accuracy.
  - First fully 4D solutions obtained using DRP and sixth order explicit differencing!
  - Stability of multiblock implementation tested (60 'blocks').

High-order 4D solutions have been obtained!

Method of Manufactured Solutions (MMS) Verification

- Given the governing equations:

$$\vec{Q}_t + \vec{E}_x + \vec{F}_y + \vec{G}_z = 0$$

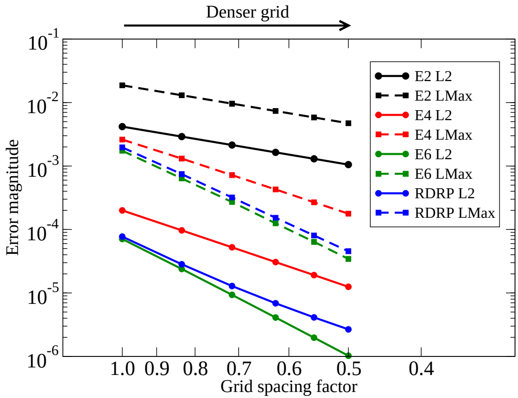
- An analytic 'Manufactured Solution' $\hat{\vec{Q}}$ is chosen and substituted into the governing equation:

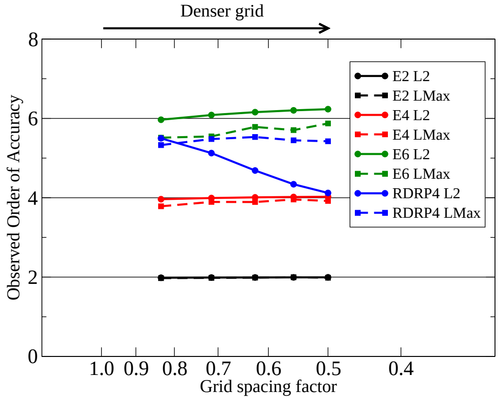
$$\hat{\vec{Q}}_t + \hat{\vec{E}}_x + \hat{\vec{F}}_y + \hat{\vec{G}}_z = \vec{S}$$

- The code is modified to add the analytic source term and used to determine the actual error and order of accuracy.

MMS provides a method to verify the 4D code!

- A series of curvilinear grids with varying grid densities are used...





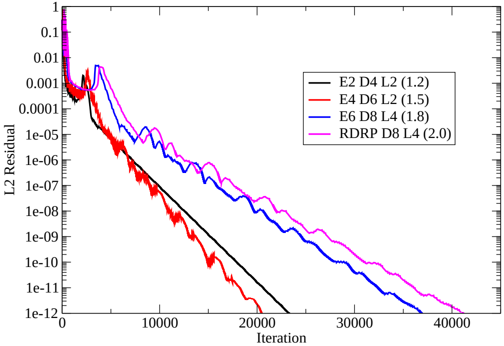
The high order methods have up to 1000x lower error!

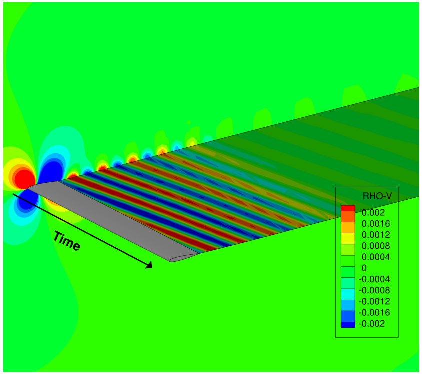
Validation Test Case: Pitching Wing of Infinite Span

- The initial validation test case is a NACA 00xx wing of infinite span pitching about the midchord location.
  - Low-speed ($M = 0.1$) inviscid flow test case, periodic in time direction.
  - NACA 00xx airfoil section with a rounded trailing edge (getting ready for fan blades!)
  - Comparing to analytic solution by Theodorsen (1935) for incompressible inviscid flow about an infinitely thin flat plate oscillating in pitch and plunge.
  - 181 x 301 x 9 x 9 single-block O-grid.
  - Thompson 1D characteristic BCs used at wall and inflow/outflow boundaries.
  - Testing showed no need for implicit boundary conditions (!)
  - Test cases were run on a Mac laptop.

First validation of the 4D STMA method against analytic solution!

- A range of reduced frequencies were run, using the optimized RDRP differencing scheme and 8^{th} order dissipation...

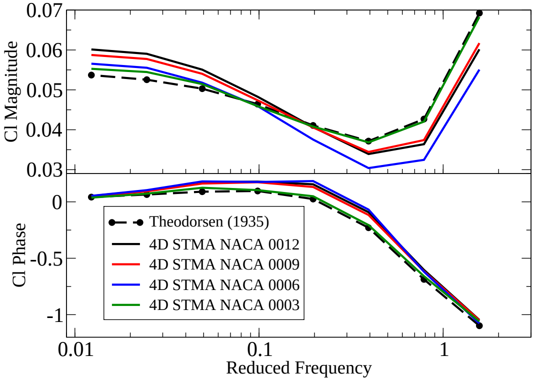




All test cases converged the L_{max} error to machine precision!

Comparison with Theory (NACA 00xx sections)

- The midspan plane is used to calculate the lift coefficient.
- The computed solution approaches the flat plate theory as airfoil thickness is reduced.



The STMA method is capturing the phase variation with 8 points per period!

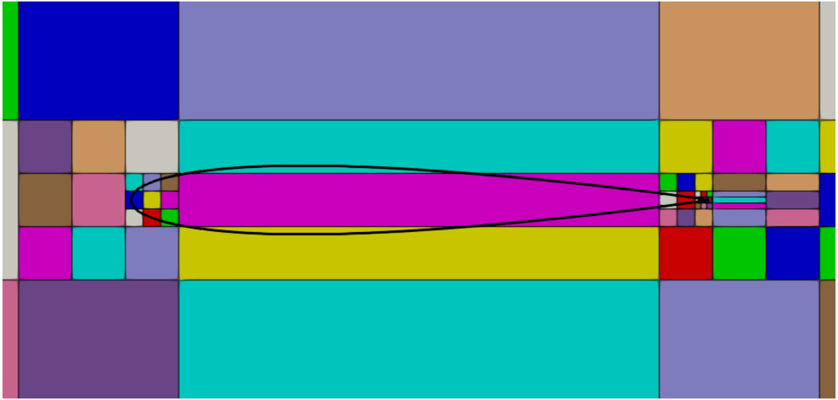
Automatic Topology Generation Methodology

- The goal is to automatically generate smooth, body-fitted multiblock curvilinear grids with full-face matching blocks for arbitrarily complex geometries in 2D, 3D, and 4D.
- The current plan for automatic grid generation takes advantage of the existing Cartesian grid generation methods to define the block topology in a robust (if not pretty!) way.
- The initial plan is that the user will provide watertight 'STL-like' unstructured definitions of the surface geometry – but the code will provide an API to interface with alternative surface definition methods.
- The 'curved PN triangles' method of Vlachos et al. (2001) is used to define Bezier surface elements and reduce the inherent faceting of an STL-like surface – giving a smoother definition of the surface for the high-order flow solver.

The result is a 'coarse-grain unstructured' body-fitted grid!

Automatic Topology Generation Methodology (2D)

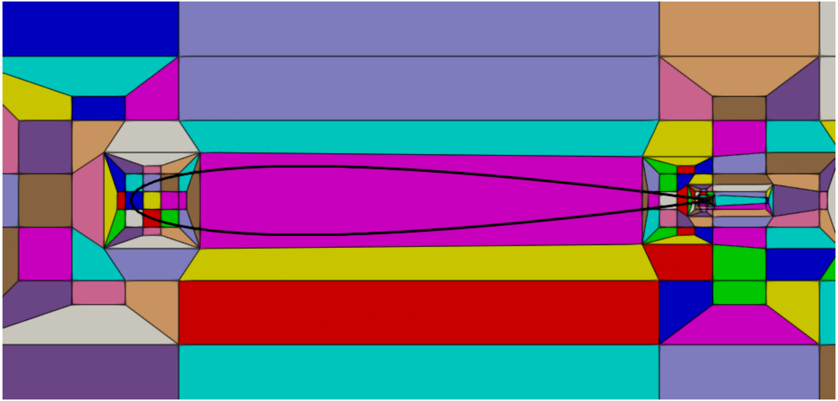
- A coarse-grain Cartesian grid method is used to define the grid blocks...



The Cartesian method 'cleanly' extends to 3D and 4D!

Automatic Topology Generation Methodology (2D)

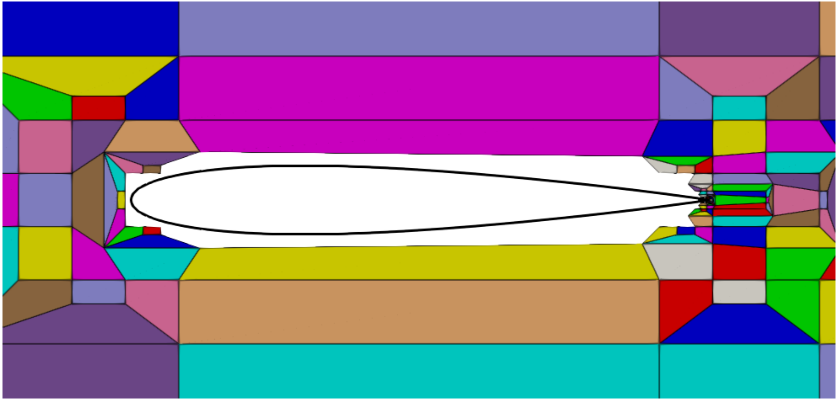
- 'Patch templates' are inserted where needed to obtain a full-face-matching grid topology...



Patch templates have been defined for 3D and 4D...

Automatic Topology Generation Methodology (2D)

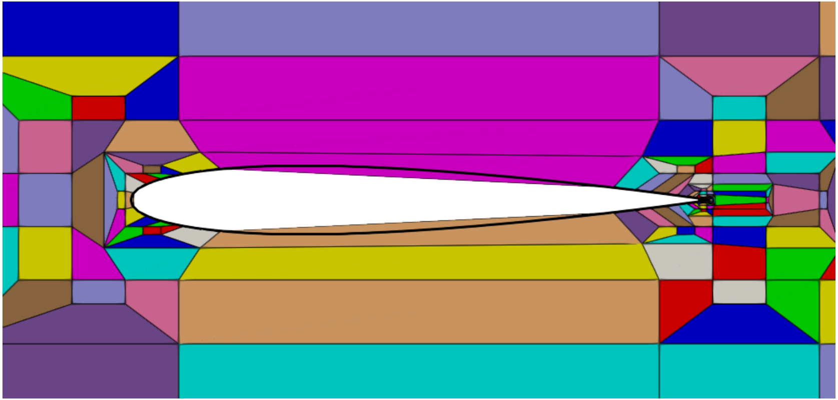
- All cells that are not completely outside the surfaces are removed...



Sharp corners on the surface need to be radiused...

Automatic Topology Generation Methodology (2D)

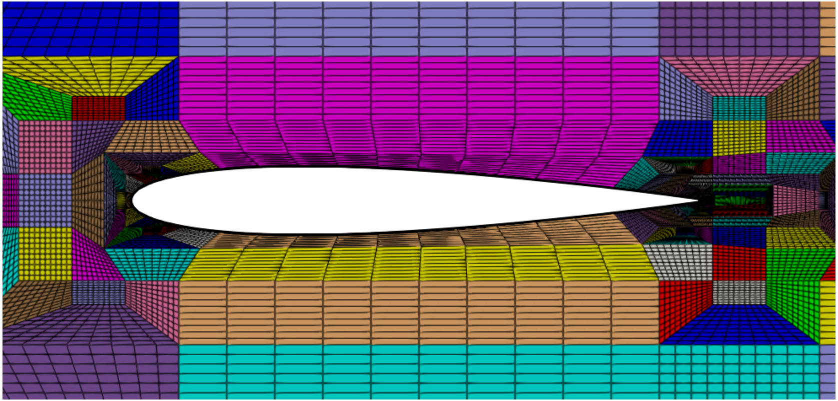
- The 'open' block faces are extruded to the surface to obtain a body-fitted topology...



The result is a body-fitted topology for arbitrary configurations!

Automatic Topology Generation Methodology (2D)

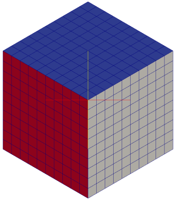
- Algebraic transfinite interpolation (TFI) is used to define the initial grid, which will be smoothed by the elliptic grid generator before use.



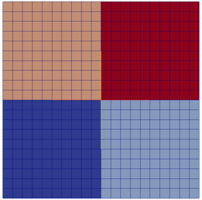
This gives a path to automatic 4D structured grid generation...

Multiblock Grid Singularities (2D)

- Structured schemes drop to 1^{st} order accuracy at grid singularities.
- These are examples of 3-way (a), 4-way (b), and 5-way (c) grid singularities at the corners of 2D blocks



a



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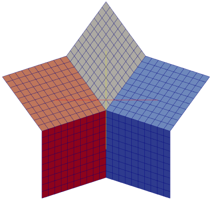
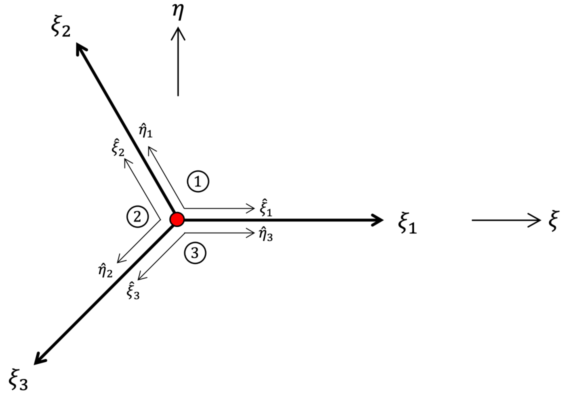


c

There are a lot of grid singularities in the automatic topologies!

The SMART Method (June 2024)

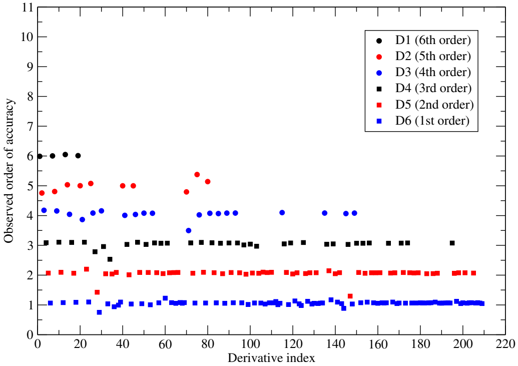
- The **S**tructured **M**apping **A**t **R**educed dimension **T**opological elements (SMART) method obtains high-order accuracy at grid singularities.

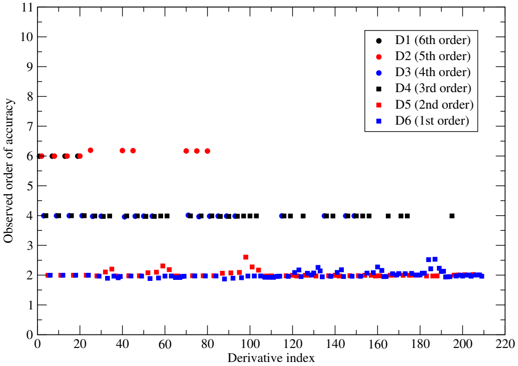


Special curvilinear mappings are defined at block boundaries...

SMART results for 6^{th} order (4D)

- These are SMART results for a 5-way singular point (left) and a 8-way nonsingular point (right) in 4D:

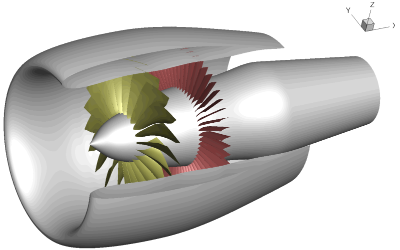




The SMART method recovers central differences for nonsingular points!

Coming Attractions

- The next steps in the process:
  - 4D automatic topology generation and improved TFI grid initialization
  - Multiblock implementation and testing of 4D grid generator and flow solver using SMART block boundaries
  - Code parallelization
  - Multigrid and automatic grid sequencing
  - Grid adaption



All objects are coded in 2D, 3D, and 4D for use in other codes...

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Questions?